Università di Bologna – Facoltà di Ingegneria Corso di Teoria dei Sistemi

MATLAB Primer

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Introduction

MATLAB is an interactive, matrix-based system for scientific and engineering calculations. You can solve complex numerical problems without actually writing a program. MATLAB is an outgrowth of the LINPACK and EISPACK projects, and has been evolving for a number of years to the current system. The name MATLAB is an abbreviation for MATrix LABoratory.

The purpose of these notes is to help you begin to use MATLAB. They can best be used handson. You are encouraged to work at the computer as you read the notes and freely experiment with examples.

You should liberally use the on-line help facility for more detailed information. After entering MATLAB as described in section 1, the command help will display a list of functions for which on-line help is available; the command help *functionname* will give information about a specific function. The command help eig, for example, will give information about the eigenvalue function eig. You can preview some of the features of MATLAB by entering the command demo.

The scope and power of MATLAB go far beyond these notes. Eventually you will want to consult the MATLAB User's Guide. Copies of the complete User's Guide are often available for review at locations such as consulting desks, terminal rooms, computing labs, and the reserve desk of the library. Consult your instructor or your local computing center to learn where the User's Guides are located at your institution.

MATLAB is available for a number of environments: Sun/Apollo/VAXstation/HP workstations, VAX, MicroVAX, Gould, PC and AT compatibles, 80386 computers, Apple Macintosh, and several parallel machines. The information in these notes applies generally to each of these environments, except as noted.

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Contents

| 1. Accessing MATLAB 1 |
|---|
| 2. Entering matrices 1 |
| 3. Matrix operations 2 |
| 4. Statements, expressions, variables 3 |
| 5. Matrix building functions 3 |
| 6. For, while, if |
| 7. Scalar functions 6 |
| 8. Vector functions |
| 9. Matrix functions 7 |
| 10. Submatrices and colon notation |
| 11. M-files |
| 12. Text strings, error messages, input 10 |
| 13. Managing M-files 10 |
| 14. Comparing efficiency of algorithms: flops and etime |
| 15. Output format |
| 16. Hard copy |
| 17. Graphics |
| 18. Reference |

1. Accessing MATLAB.

On most systems, after logging in one can enter MATLAB with the system command matlab and exit MATLAB with the command exit or quit. On a PC, for example, if properly installed, one may enter MATLAB with the command:

C> matlab

and exit it with the command:

» quit

On systems permitting multiple processes, such as a Unix system, you will find it convenient, for reasons discussed in section 13, to keep both MATLAB and your local editor active. If you are working on a workstation which runs processes in multiple windows, you will want to keep MATLAB active in one window and your local editor active in another. You may consult your instructor or your local computer center for details of the local installation.

2. Entering matrices.

MATLAB works with essentially only one kind of object—a rectangular numerical matrix with possibly complex entries; all variables represent matrices. In some situations, special meaning is attached to 1-by-1 matrices, which are scalars, and to matrices with only one row or one column, which are vectors.

Matrices can be introduced into MATLAB in several different ways:

- Entered by an explicit list of elements,
- Generated by built-in statements and functions,
- Created in M-files (see section 13 below),
- Loaded from external data files (see User's Guide).

For example, either of the statements

 $A = [1 \ 2 \ 3; \ 4 \ 5 \ 6; \ 7 \ 8 \ 9]$

and

creates the obvious 3-by-3 matrix and assigns it to a variable *A*. Try it. The elements within a row of a matrix may be separated by commas as well as a blank.

When listing a number in exponential form (e.g. 2.34e-9), blank spaces must be avoided. Listing entries of a large matrix is best done in an M-file, where errors can be easily edited away (see section 8).

The built-in functions rand, magic, and hilb, for example, provide an easy way to create matrices with which to experiment. The command rand(n), resp. rand(m,n), will create a *n*-by-*n*, resp. *m*-by-*n*, matrix with randomly generated entries; magic(n) will create an integral

_____ 1

n-by-*n* matrix which is a magic square (rows and columns have common sum); hilb(n) will create the *n*-by-*n* Hilbert matrix, the king of ill-conditioned matrices (*m* and *n* denote, of course, positive integers). Matrices can also be generated with a for-loop (see section 6 below). Try it.

Individual matrix and vector entries can be referenced with indices inside parentheses in the usual manner. For example, A(2, 3) denotes the entry in the second row and third column of a matrix A and x(3) denotes the third coordinate of a vector x. Try it.

3. Matrix operations.

The following matrix operations are available in MATLAB:

| + | addition |
|-------------|----------------|
| — | subtraction |
| * | multiplication |
| ^ | power |
| / | transpose |
| \setminus | left division |
| / | right division |

These matrix operations apply, of course, to scalars (1-by-1 matrices) as well. If the sizes of the matrices are incompatible for the matrix operation, an error message will result, except in the case of scalar-matrix operations in which case each entry of the matrix is operated on by the scalar.

The "matrix division" operations deserve special comment. If A is an invertible square matrix and b is a compatible column, resp. row, vector, then

 $x = A \setminus b$ is the solution of A * x = b and, resp.,

x = b/A is the solution of x * A = b.

In left division, if A is square, then it is factored using Gaussian elimination and these factors used to solve A * x = b. If A is not square, it is factored using Householder orthogonalization with column pivoting and the factors used to solve the under- or over- determined system in the least squares sense. Right division is defined in terms of left division by $b/A = (A' \setminus b')'$.

It is important to observe that the operations $*, \uparrow, \backslash$, and / will operate entry-wise if they are preceded by a period. For example, either $[1, 2, 3, 4] \cdot [1, 2, 3, 4]$ or $[1, 2, 3, 4] \cdot 2$ will yield [1, 4, 9, 16]. Try it.

4. Statements, expressions, and variables.

MATLAB is an *expression* language; the expressions you type are interpreted and evaluated. MATLAB statements are usually of the form

```
variable = expression, or simply expression
```

Expressions are usually composed from operators, functions, and variable names. Evaluation of the expression produces a matrix, which is then displayed on the screen and assigned to the variable for future use. If the variable name and = sign are omitted, a variable ans (for answer) is automatically created to which the result is assigned.

A statement is normally terminated with the carriage return. However, a statement can be continued to the next line with three or more periods followed by a carriage return. On the other hand, several statements can be placed on a single line if separated by commas or semicolons.

If the last character of a statement is a semicolon, the printing is suppressed, but the assignment is carried out. This is essential in suppressing unwanted printing of intermediate results.

MATLAB is case-sensitive in the names of commands, functions, and variables. For example, solveUT is not the same as solveut.

The command who will list the variables currently in the workspace. A variable can be cleared from the workspace with the command clear *variablename*. The command clear alone will clear all nonpermanent variables.

When one logs out or exits MATLAB all variables are lost. However, invoking the command save before exiting causes all variables to be written to a diskfile named matlab.mat. When one later reenters MATLAB, the command load will restore the workspace to its former state.

A runaway display or computation can be stopped on most machines without leaving MATLAB with CTRL-C (CTRL-BREAK on a PC).

The permanent variable eps (epsilon) gives the machine precision—about 10^{-16} on most machines. It is useful in determining tolerences for convergence of iterative processes.

5. Matrix building functions.

Convenient matrix building functions are

| eye | identity matrix | | |
|----------|-----------------------------------|--|--|
| zeros | matrix of zeros | | |
| ones | matrix of ones | | |
| diag | see below | | |
| triu | upper triangular part of a matrix | | |
| tril | lower triangular part of a matrix | | |
| rand | randomly generated matrix | | |
| hilb | Hilbert matrix | | |
| magic | magic square | | |
| toeplitz | see help toeplitz | | |

For example, zeros(m,n) produces an *m*-by-*n* matrix of zeros and zeros(n) produces an *n*-by-*n* one; if *A* is a matrix, then zeros(A) produces a matrix of zeros of the same size as *A*.

If x is a vector, diag(x) is the diagonal matrix with x down the diagonal; if A is a square matrix, then diag(A) is a vector consisting of the diagonal of A. What is diag(diag(A))? Try it.

Matrices can be built from blocks. For example, if A is a 3-by-3 matrix, then

B = [A, zeros(3,2); zeros(2,3), eye(2)]

will build a certain 5-by-5 matrix. Try it.

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6. For, while, if.

In their basic forms, these MATLAB flow control statements operate like those in most computer languages. For example, the statement

for i = 1:n, $x(i)=i^2$, end

or

will produce a certain *n*-vector and the statement

```
for i = n:-1:1, x(i)=i^2, end
```

will produce the same vector in reverse order. The statements

```
for i = 1:m
    for j = 1:n
        H(i, j) = 1/(i+j-1);
    end
end
H
```

will produce and print to the screen the *m*-by-*n* hilbert matrix. The semicolon on the inner statement suppresses printing of unwanted intermediate results while the last H displays the final result.

The general form of a while loop is

```
while relation
statements
end
```

The statements will be repeatedly executed as long as the relation remains true. For example, for a given number *a*, the following will compute and display the smallest nonnegative integer *n* such that $2^n \ge a$:

```
n = 0;
while 2<sup>n</sup> < a
    n = n + 1;
end
n
```

The general form of an if statement is illustrated by

```
if n < 0
    x(n) = 0;
elseif rem(n,2) == 0
    x(n) = 2;
else
    x(n) = 1;
end</pre>
```

_____ 4

In two-way branching the elseif portion would, of course, be omitted.

The relational operators in MATLAB are

| < | less than |
|----------|-----------------------|
| > | greater than |
| <= | less than or equal |
| >= | greater than or equal |
| == | equal |
| $\sim =$ | not equal. |
| | |

Note that "=" is used in an assignment statement while "==" is used in a relation. Relations may be connected or quantified by the logical operators

When applied to scalars, a relation is actually the scalar 1 or 0 depending on whether the relation is true or false. Try 3 < 5, 3 > 5, 3 == 5, and 3 == 3. When applied to matrices of the same size, a relation is a matrix of 0 and 1's giving the value of the relation between corresponding entries. Try a = rand(5), b = triu(a), a == b.

A relation between matrices is interpreted by while and if to be true if each entry of the relation matrix is nonzero. Hence, if you wish to execute *statement* when matrices A and B are equal you could type

```
if A == B
statement
end
```

but if you wish to execute *statement* when A and B are not equal, you would type

```
if any(any(A ~= B))
    statement
    end
or, more simply,
```

```
if A == B else
    statement
end
```

Note that the seemingly obvious

```
if A \sim= B, statement, end
```

will not give what is intended since *statement* would execute only if *each* of the corresponding entries of *A* and *B* differ. The functions any and all can be creatively used to reduce matrix relations to vectors or scalars. Two any's are required above since any is a vector operator (see section 8).

The for statement permits any matrix to be used instead of 1:n. See the User's Guide for details of how this feature expands the power of the for statement.

7. Scalar functions.

Certain MATLAB functions operate essentially on scalars, but operate element-wise when applied to a matrix. The most common such function are

| sin | asin | exp | abs | round |
|-----|------|-------------------|------|-------|
| cos | acos | log (natural log) | sqrt | floor |
| tan | atan | rem (remainder) | sign | ceil |

8. Vector functions.

Other MATLAB functions operate essentially on a vector (row or column), but act on an *m*-by-*n* matrix $(m \ge 2)$ in a column-by-column fashion to produce a row vector containing the results of each column. A few of these functions are

| max | sum | median | any |
|------|------|--------|-----|
| min | prod | mean | all |
| sort | | std | |

For example, the maximum entry in a matrix A is given by max(max(A)) rather than max(A). Try it.

9. Matrix functions.

Much of MATLAB's power comes from its matrix functions. The most useful ones are

| eig | eigenvalues and eigenvectors |
|-------|--|
| chol | cholesky factorization |
| svd | singular value decomposition |
| inv | inverse |
| lu | LU factorization |
| qr | QR factorization |
| hess | hessenberg form |
| schur | schur decomposition |
| rref | reduced row echelon form |
| expm | matrix exponential |
| sqrtm | matrix square root |
| poly | characteristic polynomial |
| det | determinant |
| size | size |
| norm | 1-norm, 2-norm, F-norm, ∞ -norm |
| cond | condition number in the 2-norm |
| rank | rank |

MATLAB functions may have single or multiple output arguments. For example,

y = eig(A), or simply eig(A)

produces a column vector containing the eigenvalues of A while

[U,D] = eig(A)

produces a matrix U whose columns are the eigenvectors of A and a diagonal matrix D with the eigenvalues of A on its diagonal. Try it.

10. Submatrices and colon notation.

Vectors and submatrices are used often in MATLAB to acheive fairly complex data manipulation effects. "Colon notation", which is used both to generate vectors and reference submatrices, and subscripting by vectors are keys to efficient manipulation of these objects. Creative use of these features permits one to minimize the use of loops (which slows down MATLAB) and to make code simple and readable. *Special effort should be made to become familiar with them*.

The expression 1:5 (met earlier in for statements) is actually the row vector $[1 \ 2 \ 3 \ 4 \ 5]$. The numbers need not be integers nor the increment one. For example,

0.2:0.2:1.2 gives [0.2 0.4 0.6 0.8 1.0 1.2], and 5:-1:1 gives [5 4 3 2 1].

The following statements will, for example, generate a table of sines (try it):

x = [0.0:0.1:2.0]'; y = sin(x); [x y]

Note that since sin operates entry-wise, it produces a vector y from the vector x.

The colon notation can be used to access submatrices of a matrix. For example,

A(1:4,3) is the column vector consisting of the first four entries of the third column of A.

A colon by itself denotes an entire row or column:

A(:, 3) is the third column of A, and A(1:4, :) is the first four rows.

Arbitrary integral vectors can be used as subscripts:

 $A(:, [2 \ 4])$ contains as columns, columns 2 and 4 of A.

Such subscripting can be used on both sides of an assignment statement:

 $A(:, [2 \ 4 \ 5]) = B(:, 1:3)$ replaces columns 2,4,5 of A with the first three columns of B. Note that the *entire* altered matrix A is printed and assigned. Try it.

Columns 2 and 4 of A can be multiplied on the right by the 2-by-2 matrix [1 2;3 4] by:

A(:,[2,4]) = A(:,[2,4])*[1 2;3 4]

Once again, the entire altered matrix is printed and assigned.

If x is an *n*-vector, what is the effect of the statement x = x(n:-1:1)? Try it.

To appreciate the usefulness of these features, compare these MATLAB statements with a Pascal, FORTRAN, or C routine to effect the same.

11. M-files.

MATLAB can execute a sequence of statements stored on diskfiles. Such files are called "M-files" because they must have the file type of ".m" as the last part of their filename. Much of your work with MATLAB will be in creating and refining M-files.

There are two types of M-files: *script files* and *function files*. A script file consists of a sequence of normal MATLAB statements. If the file has the filename, say, rotate.m, then the MATLAB command rotate will cause the statements in the file to be executed. Variables in a script file are global and will change the value of the environment.

Script files are often used to enter data into a large matrix; in such a file, entry errors can be easily edited out. If, for example, one enters in a diskfile data.m

A = [1 2 3 4 5 6 7 8];

then the MATLAB statement data will cause the assignment given in data.m to be carried out.

An M-file can reference other M-files, including referencing itself recursively.

Function files provide extensibility to MATLAB. You can create new functions specific to your problem which will then have the same status as other MATLAB functions. Variables in a function file are local.

We first illustrate with a simple example of a function file:

```
function P = prodsqr(A,B)
% PRODSQR Product of the square of two matrices.
P = A^2*B^2;
```

Memory will be used more efficiently if *A* is overwritten with the result:

```
function A = prodsqr(A,B)
% PRODSQR Product of the square of two matrices.
A = A<sup>2</sup>*B<sup>2</sup>;
```

This should be placed in a diskfile with filename prodsqr.m (corresponding to the function name). The first line declares the function name, input arguments, and output arguments; without this line the file would be a script file. Then a MATLAB statement

z = prodsqr(x, y), for example, will cause the variables x and y to be passed to the variables A and B in the function file with the output result being passed out to the variable z. Since variables in a function file are local, their names are independent of those in the environment.

A function may also have multiple output arguments. For example:

function [mean, stdev] = stat(x)
% STAT Mean and standard deviation
% For a vector x, stat(x) returns the
% mean and standard deviation of x.
% For a matrix x, stat(x) returns two row vectors
% containing, respectively, the mean and standard
% deviation of each column.

```
[m n] = size(x);
if m == 1
  m = n; % handle case of a row vector
end
mean = sum(x)/m;
stdev = sqrt(sum(x.^2)/m - mean.^2)
```

Once this is placed in a diskfile stat.m, a MATLAB command [xm, xd] = stat(x), for example, will assign the mean and standard deviation of the entries in the vector x to xm and xd, respectively. Single assignments can also be made with a function having multiple output arguments. For example, xm = stat(x) will assign the mean of x to xm.

This function illustrates some of the MATLAB features that can be used to produce efficient code. Note, for example, that x. 2 is the matrix of squares of the entries of x, that sum is a vector function (section 6), that sqrt is a scalar function (section 5), and that the division in sum(x)/m is a matrix-scalar operation.

The % symbol indicates that the rest of the line is a comment; MATLAB will ignore the rest of the line. However, the first few comment lines, which document the M-file, are available to the on-line help facility and will be displayed if, for example, help stat is entered.

12. Text strings, error messages, input.

Text strings are entered into MATLAB surrounded by single quotes. For example,

s = 'This is a test'

assigns the given text string to the variable s.

Text strings can be displayed with the function disp. For example:

```
disp('this message is hereby displayed')
```

Error messages are best displayed with the function error

error('Sorry, the matrix must be symmetric')

since it causes execution to exit the M-file.

In an M-file the user can be prompted to interactively enter input data with the function input. When, for example, the statement

iter = input('Enter the number of iterations: ')

is encountered, the prompt message is displayed and execution pauses while the user keys in the input data. Upon pressing the return key, the data is assigned to the variable *iter* and execution resumes.

13. Managing M-files.

While using MATLAB one frequently wishes to create or edit an M-file and then return to MATLAB. One wishes to keep MATLAB active while editing a file since otherwise all variables would be lost upon exiting.

This can be easily done using the !-feature. If, while in MATLAB, you precede it with an !, any system command—such as those for editing, printing, or copying a file—can be executed without

exiting MATLAB. If, for example, the system command ed accesses your editor, the MATLAB command

» !ed rotate.m

will let you edit the file named rotate.m using your local editor. Upon leaving the editor, you will be returned to MATLAB just where you left it.

As noted in section 1, on systems permitting multiple processes, such as one running Unix, it may be preferable to keep both MATLAB and your local editor active, keeping one process suspended while working in the other. If these processes can be run in multiple windows, as on a Sun workstation, you will want to keep MATLAB active in one window and your editor active in another.

You may consult your instructor or your local computing center for details of the local installation.

When in MATLAB, the command dir will list the contents of the current directory while the command what will list only the M-files in the directory. The MATLAB commands delete and type can be used to delete a diskfile and print a file to the screen, respectively, and chdir can be used to change the working directory. While these commands may reflect system commands, they avoid the use of an !.

14. Comparing efficiency of algorithms: flops and etime.

Two measures of the efficiency of an algorithm are the number of floating point operations (flops) performed and the elapsed time.

The MATLAB function flops keeps a running total of the flops performed. The command flops(0) will reset flops to 0. Hence entering flops(0) immediately before executing an algorithm and flops immediately after gives the flop count for the algorithm.

The MATLAB function clock gives the current time accurate to a hundreth of a second (see help clock). Given two such times t1 and t2, etime(t2,t1) gives the elapsed time from t1 to t2. One can, for example, measure the time required to solve a given linear system Ax = b using Gaussian elimination as follows:

time = clock; x = A\b; time = etime(clock,time)

You may wish to compare this time—and flop count—with that for solving the system using x = inv(A) *bi. Try it.

It should be noted that, on timesharing machines such as the VAX, etime is not a reliable measure of the efficiency of an algorithm since the rate of execution depends on how busy the computer is at the time.

15. Output format.

While all computations in MATLAB are performed in double precision, the format of the displayed output can be controlled by the following commands.

| format short | fixed point with 4 decimal places (the default) |
|----------------|---|
| format long | fixed point with 14 decimal places |
| format short e | scientific notation with 4 decimal places |
| format long e | scientific notation with 15 decimal places |

Once invoked, the chosen format remains in effect until changed.

The command format compact will suppress most blank lines allowing more information to be placed on the screen or page. It is independent of the other format commands.

16. Hardcopy.

Hardcopy is most easily obtained with the diary command. The command

diary filename

causes what appears subsequently on the screen (except graphics) to be written on the named diskfile (if the filename is omitted it will be written to a default file named diary) until one gives the command diary off; the command diary on will cause writing to the file to resume, etc. When finished, you can edit the file as desired and print it out on the local system. The !-feature (see section 13) will permit you to print the file without leaving MATLAB.

17. Graphics.

MATLAB has excellent capability to produce planar plots and 3-D mesh surface plots. To see some of these capabilities, enter the command plotdemo.

Not all terminals for a mainframe computer support graphics. If you experience difficulty, check with your local system manager. If, however, you have terminal emulation software for your PC which supports Tektronix 4010 graphics, you may be able to explore this on the mainframe via dial-up.

Graphics are, however, supported on workstations such as the Sun and on a PC with the appropriate graphics hardware and software.

The plot command creates linear x-y plots; if x and y are vectors of the same length, the command plot(x, y) opens a graph window and draws an x-y plot of the elements of x versus the elements of y. You can, for example, draw the graph of the sine function over the interval -4 to 4 with the following commands:

x = -4:.01:4; y = sin(x); plot(x,y)

Try it. The vector x is a partition of the domain with meshsize 0.01 while y is a vector giving the values of sine at the nodes of this partition (recall that sin operates entrywise – see section 7).

When in the graph screen, pressing any key will return you to the command screen while the command shg (show graph) will then return you to the current graph screen. When creating graphics on, for example, a Sun, you may wish to keep the graph window exposed—but moved to the side—and the command window active.

As a second example, you can draw the graph of $y = e^{-x^2}$ over the interval -1.5 to 1.5 as follows:

```
x = -1.5:.01:1.5;
y = exp(-x.^2);
plot(x,y)
```

Note that one must precede $\hat{}$ by a period to ensure that it operates entrywise (see section 3).

You are referred to the User's Guide or the help facility for such features as multiple plots, titles, labels, gridlines, and manual scaling.

A hardcopy of the graph window can be obtained with the MATLAB command print:

» print

Three dimensional mesh surface plots are drawn with the function mesh. The command mesh(z) creates a three dimensional perspective plot of the elements of the matrix z. The mesh surface is defined by the z-coordinates of points above a rectangular grid in the x-y plane. Try mesh(eye(10)).

To draw the graph of a function z = f(x, y) over a rectangle, one first defines vectors xx and yy which give partitions of the sides of the rectangle. With the function meshdom (mesh domain) one then creates a matrix x, each row of which equals xx and whose column length is the length of yy, and similarly a matrix y, each column of which equals yy, as follows:

[x,y] = meshdom(xx,yy);

One then computes a matrix z, obtained by evaluating f entrywise over the matrices x and y, to which mesh can be applied.

You can, for example, draw the graph of $z = e^{-x^2 - y^2}$ over the square $[-2, 2] \times [-2, 2]$ as follows (try it):

```
xx = -2:.1:2;
yy = xx;
[x,y] = meshdom(xx,yy);
z = exp(-x.^2 - y.^2);
mesh(z)
```

One could, of course, replace the first three lines of the preceding with

[x,y] = meshdom(-2:.1:2, -2:.1:2);

You are referred to the User's Guide for further details regarding mesh.

18. Reference.

There are many MATLAB features which cannot be included in these introductory notes. Listed below are some of the MATLAB functions and operators available, grouped by subject area¹. Use the on-line help facility or consult the User's Guide for more detailed information on the functions.

There are many functions beyond these. There exist, in particular, several "toolboxes" of functions for specific areas; included among such are signal processing, control theory, dynamical systems(system identification), and chemometrics². These can be explored via the command help.

¹ Source: MATLAB User's Guide, version 3.5

 $^{^2}$ The toolboxes, which are optional, may not be installed on your system.

| | General |
|----------|---------------------------|
| help | help facility |
| demo | run demonstrations |
| who | list variables in memory |
| what | list M-files on disk |
| size | row and column dimensions |
| length | vector length |
| clear | clear workspace |
| computer | type of computer |
| ^C | local abort |
| exit | exit MATLAB |
| quit | same as exit |

| Matrix Operators | | Ar | ray Operators |
|------------------|---------------------|----|----------------|
| + | addition | + | addition |
| — | subtraction | _ | subtraction |
| * | multiplication | .* | multiplication |
| / | right division | ./ | right division |
| \setminus | left division | .\ | left division |
| ^ | power | | power |
| , | conjugate transpose | .' | transpose |

| Relational and Logical Operators | | | |
|----------------------------------|-----------------------|--------|-----|
| < | less than | & | and |
| <= | less than or equal | | or |
| > | greater than | \sim | not |
| >= | greater than or equal | | |
| == | equal | | |
| \sim_{\equiv} | not equal | | |

| | Special Characters |
|---|--|
| = | assignment statement |
| [| used to form vectors and matrices |
|] | see [|
| (| arithmetic expression precedence |
|) | see (|
| | decimal point |
| | continue statement to next line |
| , | separate subscripts and function arguments |
| ; | end rows, suppress printing |
| % | comments |
| : | subscripting, vector generation |
| ! | execute operating system command |

| Special Values | | |
|----------------|-------------------------------------|--|
| ans | answer when expression not assigned | |
| eps | floating point precision | |
| pi | π | |
| i, j | $\sqrt{-1}$ | |
| inf | ∞ | |
| NaN | Not-a-Number | |
| clock | wall clock | |
| date | date | |
| flops | floating point operation count | |
| nargin | number of function input arguments | |
| nargout | number of function output arguments | |

| D' 1 D'1 | | | |
|----------|----------------------------|--|--|
| | Disk Files | | |
| chdir | change current directory | | |
| delete | delete file | | |
| diary | diary of the session | | |
| dir | directory of files on disk | | |
| load | load variables from file | | |
| save | save variables to file | | |
| type | list function or file | | |
| what | show M-files on disk | | |
| fprintf | write to a file | | |
| pack | compact memory via save | | |

| | Special Matrices |
|----------|--------------------------------|
| compan | companion |
| diag | diagonal |
| eye | identity |
| gallery | esoteric |
| hadamard | Hadamard |
| hankel | Hankel |
| hilb | Hilbert |
| invhilb | inverse Hilbert |
| linspace | linearly spaced vectors |
| logspace | logarithmically spaced vectors |
| magic | magic square |
| meshdom | domain for mesh points |
| ones | constant |
| rand | random elements |
| toeplitz | Toeplitz |
| vander | Vandermonde |
| zeros | zero |

| Matrix Manipulation | | |
|---------------------|--|--|
| rot90 | rotation | |
| fliplr | flip matrix left-to-right | |
| flipud | flip matrix up-to-down | |
| diag | diagonal matrices | |
| tril | lower triangular part | |
| triu | upper triangular part | |
| reshape | reshape | |
| .' | transpose | |
| : | convert matrix to single column; A (:) | |

| Relational and Logical Functions | |
|----------------------------------|--------------------------------------|
| any | logical conditions |
| all | logical conditions |
| find | find array indices of logical values |
| isnan | detect NaNs |
| finite | detect infinities |
| isempty | detect empty matrices |
| isstr | detect string variables |
| strcomp | compare string variables |

| Control Flow | | |
|--------------|-------------------------------------|--|
| if | conditionally execute statements | |
| elseif | used with if | |
| else | used with if | |
| end | terminate if, for, while | |
| for | repeat statements a number of times | |
| while | do while | |
| break | break out of for and while loops | |
| return | return from functions | |
| pause | pause until key pressed | |

| Programming and M-Files | | |
|-------------------------|-----------------------------------|--|
| input | get numbers from keyboard | |
| keyboard | call keyboard as M-file | |
| error | display error message | |
| function | define function | |
| eval | interpret text in variables | |
| feval | evaluate function given by string | |
| echo | enable command echoing | |
| exist | check if variables exist | |
| casesen | set case sensitivity | |
| global | define global variables | |
| startup | startup M-file | |
| getenv | get environment string | |
| menu | select item from menu | |
| etime | elapsed time | |
| | | |

| Text and Strings | | |
|--|--|--|
| convert string to ASCII values | | |
| evaluate text macro | | |
| convert number to string | | |
| convert integer to string | | |
| set flag indicating matrix is a string | | |
| convert number to string | | |
| detect string variables | | |
| compare string variables | | |
| convert hex string to number | | |
| | | |

| Command Window | | |
|----------------|---------------------------|--|
| clc | clear command screen | |
| home | move cursor home | |
| format | set output display format | |
| disp | display matrix or text | |
| fprintf | print formatted number | |
| echo | enable command echoing | |

| | Graph Paper |
|----------|----------------------------|
| plot | linear X-Y plot |
| loglog | loglog X-Y plot |
| semilogx | semi-log X-Y plot |
| semilogy | semi-log X-Y plot |
| polar | polar plot |
| mesh | 3-dimensional mesh surface |
| contour | contour plot |
| meshdom | domain for mesh plots |
| bar | bar charts |
| stairs | stairstep graph |
| errorbar | add error bars |
| | |

| Graph Annotation | | |
|------------------|---------------------------|--|
| title | plot title | |
| xlabel | x-axis label | |
| ylabel | y-axis label | |
| grid | draw grid lines | |
| text | arbitrarily position text | |
| gtext | mouse-positioned text | |
| ginput | graphics input | |

| Graph Window Control | | |
|----------------------|---------------------|--|
| axis | manual axis scaling | |
| hold | hold plot on screen | |
| shg | show graph window | |
| clg | clear graph window | |
| subplot | split graph window | |

| Graph Window Hardcopy | |
|-----------------------|-----------------------|
| print | send graph to printer |
| prtsc | screen dump |
| meta | graphics metafile |

| | Elementary Math Functions |
|-------|-------------------------------------|
| abs | absolute value or complex magnitude |
| angle | phase angle |
| sqrt | square root |
| real | real part |
| imag | imaginary part |
| conj | complex conjugate |
| round | round to nearest integer |
| fix | round toward zero |
| floor | round toward $-\infty$ |
| ceil | round toward ∞ |
| sign | signum function |
| rem | remainder |
| exp | exponential base e |
| log | natural logarithm |
| log10 | log base 10 |

| Trigonometric Functions | |
|-------------------------|--------------------------|
| sin | sine |
| cos | cosine |
| tan | tangent |
| asin | arcsine |
| acos | arccosine |
| atan | arctangent |
| atan2 | four quadrant arctangent |
| sinh | hyperbolic sine |
| cosh | hyperbolic cosine |
| tanh | hyperbolic tangent |
| asinh | hyperbolic arcsine |
| acosh | hyperbolic arccosine |
| atanh | hyperbolic arctangent |

| Special Functions | |
|-------------------|--|
| bessel | bessel function |
| gamma | gamma function |
| rat | rational approximation |
| erf | error function |
| inverf | inverse error function |
| ellipk | complete elliptic integral of first kind |
| ellipj | Jacobian elliptic integral |

| Decompositions and Factorizations | | |
|-----------------------------------|---|--|
| balance | balanced form | |
| backsub | backsubstitution | |
| cdf2rdf | convert complex-diagonal to real-diagonal | |
| chol | Cholesky factorization | |
| eig | eigenvalues and eigenvectors | |
| hess | Hessenberg form | |
| inv | inverse | |
| lu | factors from Gaussian elimination | |
| nnls | nonnegative least squares | |
| null | null space | |
| orth | orthogonalization | |
| pinv | pseudoinverse | |
| qr | orthogonal-triangular decomposition | |
| qz | QZ algorithm | |
| rref | reduced row echelon form | |
| schur | Schur decomposition | |
| svd | singular value decomposition | |

| | Matrix Conditioning |
|-------|--------------------------------------|
| cond | condition number in 2-norm |
| norm | 1-norm,2-norm,F-norm, ∞ -norm |
| rank | rank |
| rcond | condition estimate (reciprocal) |

| Elementary Matrix Functions | | |
|-----------------------------|---------------------------|--|
| expm | matrix exponential | |
| logm | matrix logarithm | |
| sqrtm | matrix square root | |
| funm | arbitrary matrix function | |
| poly | characteristic polynomial | |
| det | determinant | |
| trace | trace | |
| kron | Kronecker tensor product | |

| Polynomials | | |
|-------------|--|--|
| poly | characteristic polynomial | |
| roots | polynomial roots-companion matrix method | |
| roots1 | polynomial roots—Laguerre's method | |
| polyval | polynomial evaluation | |
| polyvalm | matrix polynomial evaluation | |
| conv | multiplication | |
| deconv | division | |
| residue | partial-fraction expansion | |
| polyfit | polynomial curve fitting | |

| Column-wise Data Analysis | |
|---------------------------|--------------------------------|
| max | maximum value |
| min | minimum value |
| mean | mean value |
| median | median value |
| std | standard deviation |
| sort | sorting |
| sum | sum of elements |
| prod | product of elements |
| cumsum | cumulative sum of elements |
| cumprod | cumulative product of elements |
| diff | approximate derivatives |
| hist | histograms |
| corrcoef | correlation coefficients |
| cov | covariance matrix |
| cplxpair | reorder into complex pairs |

| | Signal Processing |
|----------|--------------------------------|
| abs | complex magnitude |
| angle | phase angle |
| conv | convolution |
| corrcoef | correlation coefficients |
| cov | covariance |
| deconv | deconvolution |
| fft | radix-2 fast Fourier transform |
| fft2 | two-dimensional FFT |
| ifft | inverse fast Fourier transform |
| ifft2 | inverse 2-D FFT |
| fftshift | FFT rearrangement |

| Interpolation | |
|---------------|-------------------|
| spline | cubic spline |
| table1 | 1-D table look-up |
| table2 | 2-D table look-up |

| | Differential Equation Solution |
|-------|---|
| ode23 | 2nd/3rd order Runge-Kutta method |
| ode45 | 4th/5th order Runge-Kutta-Fehlberg method |

| Nonlinear Equations and Optimization | |
|--------------------------------------|---|
| fmin | minimum of a function of one variable |
| fmins | minimum of a multivariable function |
| fsolve | solution of a system of nonlinear equations |
| | (zeros of a multivariable function) |
| fzero | zero of a function of one variable |